

Modified non-local gravity

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Abstract

In this note I provide an extended version of the talk given at BW2011 workshop. The concise introduction to the non-local SFT motivated models is given with an emphasis on the non-local generalization of gravity. A number of open questions and future directions in the development of such models is outlined.

1 Introduction

String-inspired non-local field theory models have gained popularity in recent years. These models contain an infinite series of higher derivative terms in their action (and hence the term “non-local”¹) similar to what one finds in the action of the tachyon in string field theory (SFT) [1], in stringy toy models such as p-adic string models [2] or strings on random lattice [3]. There are several motivations for studying such field theories. These theories often present novel cosmological features, and has been used to obtain inflation, dark energy, and bounce. Like most other higher derivative models, these field theories are better behaved in the UV as compared to the usual renormalizable models, the quantum theory, in fact, is finite and provides deviations in scattering cross-sections which may be detectable in particle physics experiments. Their study can provide insight into the “real” string theory, for instance the existence of thermal duality, previously only conjectured using stringy arguments, was explicitly demonstrated. Furthermore even though one has an infinite series of higher derivative terms, these non-local models can avoid the problem of ghosts by avoiding additional poles in the propagator, and thus admit the possibility of a consistent UV completion.

Although mostly non-local scalar field theories were explored, in this note I mainly focus on the non-local modifications of gravity that was essentially proposed to see whether the big bang problem gets resolved in these models. While a specific exact nonsingular bouncing solution was indeed discovered in [4], in [5] it was further shown that such bouncing solutions exist for a large class of these non-local models, and that a subset of these theories admit for a late-time de Sitter expansion (we also note works [6] which discuss related models as well). This

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¹The field equations can also often be cast as integral equations, a signature of non-local theories.

therefore provides us with a natural possibility of constructing a past geodesically complete model of inflation where the inflationary phase is preceded by a contracting phase and a nonsingular bounce. In this context we note that it has been known for a while that the standard near exponentially expanding space-time is not past geodesically complete.

I will start with a very brief introduction in the non-local SFT motivated models and then proceed with more involved non-local set-ups.

2 Non-local models basics

Non-local scalar field action can be easily seen in the computation in SFT. Consider cubic SFT which may be either bosonic or fermionic. Performing string world-sheet calculations in the SFT action one arrives to the action for space-time fields. This action consists of two parts: the quadratic form which is the kinetic part and the cubic interaction among all the fields. Schematically we can write it as

$$S = \int d^D x \sqrt{-\eta} \left(\frac{1}{2} \phi_i K_{ij}(\square) \phi_j - v_{ijk}(e^{-\frac{\beta}{2}\square} \phi_i)(e^{-\frac{\beta}{2}\square} \phi_j)(e^{-\frac{\beta}{2}\square} \phi_k) \right) \quad (1)$$

where D is the dimension in which our theory lives, β is a parameter determined exclusively by the conformal field theory and \square is the d'Alambertian operator computed on the space-time metric. It is natural to expect $\beta < 0$ corresponding to a convergent propagator at large momenta.

Here we see the very important feature: we have explicit non-locality in the action. We stress here that this is general feature of the SFT based models². Without the interaction term action (1) is the free string spectrum and if everything is correct with SFT then the spectrum coincides with the first quantized string theory. For cubic SFT this is the case. The form K_{ij} is the first degree polynomial in the box, meaning it has no higher derivatives. This is a very important statement saying that our theory has no extra excitations.

If only one field is relevant (like in the tachyon condensation scenarios) one can quite familiarly analyze properties of a potential. Indeed, defining the potential as minus action evaluated at $\square = 0$ one can study its vacuum structure. What is novel and quite non-trivial is the dynamics even for linearized models.

Action with one scalar field looks like

$$S = \int d^D x \sqrt{-\eta} \left(\frac{1}{2} \phi(\square - m^2) \phi - v(\square, \phi) \right) \quad (2)$$

where v as before is not necessarily the cubic monomial in ϕ and without quadratic in ϕ term. Suppose there are two distinct vacua, say $\phi = 0$ and $\phi = \phi_0$. Perturbative vacuum has only one excitation with $-k^2 = m^2$. Physics in the non-perturbative vacuum $\phi = \phi_0$ is transparent after the shift $\phi = \phi_0 + \chi$. The quadratic part becomes

$$S_0 = \int d^D x \sqrt{-\eta} \left(\frac{1}{2} \chi(\square - m^2) \chi - \frac{\lambda}{2} \chi \mathcal{G}(\square) \chi \right) = \int d^D x \sqrt{-\eta} \frac{1}{2} \chi \mathcal{F}_\chi(\square) \chi \quad (3)$$

²Appearance of higher derivatives is not unique for this theory. Non-commutative theories, for instance, also have higher derivative, but these non-local structures are very different.

where we integrated by parts to move the box. $\mathcal{G}(\square)$ is not obviously expected to be an analytic function and unlikely just an exponent. With the general function $\mathcal{G}(\square)$ it is possible that function $\mathcal{F}_\chi(\square)$ has finite number of poles (giving finite number of ghosts) or just no ghost or something else.

Indeed, if $\mathcal{G}(\square) = (\epsilon\square - m_0^2)e^g(\square) - \square + m^2$ with $g(\square)$ an entire function we end up with $\mathcal{F}(\square) = (\epsilon\square - m_0^2)e^g(\square)$. This produces a single massive excitation which is either ordinary field or ghost depending on the sign of ϵ .

The new feature is that physics in different vacua can be completely different in contrast to canonical field theories where at most masses get shifted. For example, there may be different number and nature of states in distinct vacua. The more comprehensive analysis can be found in, for instance, [7] and references therein.

3 Non-local gravity

In fact nothing restricts us to scalar fields only and one can build models with other fields involving towers of d'Alembertian operators. We therefore proceed to the non-local gravity.

The non-local modification of the Einstein gravity, which is proposed in [4] is described by the following action:

$$S = \int d^4x \sqrt{-g} \left(\frac{M_P^2}{2} R + \frac{1}{2} R \mathcal{F}(\square/M_*^2) R - \Lambda \right) \quad (4)$$

where M_P is the Planck mass: $M_P^2 = 1/(8\pi G_N)$, G_N is the Newtonian gravitational constant, M_* is the mass scale at which the higher derivative terms in the action become important. An analytic function $\mathcal{F}(\square/M_*^2) = \sum_{n \geq 0} f_n \square^n$ is an

ingredient inspired by the SFT. The operator \square is the covariant d'Alembertian. In the case of an infinite series we have a non-local action.

Such a model exhibits a number of interesting properties outlined in the introduction for the non-local modifications of gravity. Namely, one can construct solutions to the equations of motion which describe a non-singular bounce and/or de Sitter late time attractor. The surprising point is that exact analytic solutions can be found.

On the other hand an intriguing connection with p -adic like models arises if one introduces a scalar field in order to rewrite the action. Indeed, one can check that the following equivalent action can be written:

$$S = \int d^4x \sqrt{-g} \left(\frac{M_P^2}{2} R \left(1 + \frac{2}{M_P^2} \psi \right) - \frac{1}{2} \psi \mathcal{G}(\square/M_*^2) \psi - \Lambda \right) \quad (5)$$

Varying over ψ and substituting the resulting solution of the corresponding equation of motion back in the action restores (4) provided $\mathcal{G} = \mathcal{F}^{-1}$.

Further one can make the conformal transformation and rescale the metric such that there is no non-minimal coupling. This is however not as smooth as in local theories because the box is the covariant box and the rescaling of the metric will introduce non-trivial dependence on the scalar field inside the function \mathcal{F} . It still would be feasible to analyze the linearized theories around a given vacuum but the whole model would look much more cumbersome.

The above quadratic in scalar curvature actions without the cosmological constant are almost enough to study the Minkowski background³. Indeed, higher curvature terms would vanish both in the background and in the linear variation of equations of motion producing no new effects.

One can check, however, that in order to have the de Sitter solution one must keep the cosmological constant explicitly. It is easy to understand since de Sitter metric produces covariantly constant Riemann tensor and therefore higher derivatives are not involved while it is known that R^2 theories of gravity do not have the de Sitter solution. To overcome this higher degrees of curvature may be introduced in the action. This can make it possible to have the de Sitter Universe without having the cosmological term explicitly.

The major and significant difference with well known modified theories of gravity is the fact that higher derivative structures can eliminate unwanted (ghost) excitations keeping the theory well defined.

4 Simplest proposal to avoid the cosmological constant

As one of the simplest proposal we promote action (4) to the following form

$$S = \int d^4x \sqrt{-g} f(\Pi(\Box/M_\star^2)R) \quad (6)$$

It is clearly just action (4) if f is the quadratic polynomial, $\Pi^2 = \mathcal{F}$ and constants are adjusted properly. Again, almost as in usual $f(R)$ gravity theories one can pass to the kind of scalar-tensor theory which is now non-local though.

Canonically in local modified gravity theories

$$S = \int d^4x \sqrt{-g} f(R) \rightarrow S = \int d^4x \sqrt{-g} (R f'(\psi) - \psi f'(\psi) + f(\psi))$$

where prime denotes the derivative w.r.t. the argument. In case of non-local theories we just would have

$$S = \int d^4x \sqrt{-g} ((\Pi R) f'(\psi) - \psi f'(\psi) + f(\psi))$$

Further one can move the operator Π from R to $f'(\psi)$ and make the field redefinition $\Pi f'(\psi) = \chi$. At this stage the action is again similar to p -adic string theory. One more transformation, which is the metric rescaling is possible to separate explicitly the scalar curvature. In this model de Sitter solution exists provided there is a non-trivial vacuum for the field χ .

The next question whether such an action may have again the non-singular bounce as well, is there a solution which starts with the bounce and ends up in the de Sitter phase and then one may ask whether perturbations behave good or not.

This is surely a highly non-trivial thing to construct at least one solution. The only hope so far is to make use of some ansatz. In case of the quadratic action (4) the following ansatz does work

$$\Box R = r_1 R + r_2 \quad (7)$$

³See [8] for the most general non-local action suitable for the Minkowski space-time.

One can check that specializing to the space-homogeneous and isotropic spatially flat FRW 4-dimensional Universe with the metric $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$ the following scale factors satisfy the above ansatz

$$a(t) = a_0 \cosh(\lambda t) \text{ and } a(t) = a_0 e^{\frac{\lambda}{2} t^2}$$

In order for this functions become solutions one has to adjust coefficients of the Taylor expansion of operator function \mathcal{F} (or Π). For the quadratic action only two conditions on values of $\mathcal{F}(r_1)$ and $\mathcal{F}'(r_1)$ arise. At present stage it is an open question to propagate this idea to action (5) since equations become more complicated.

5 Beyond scalar curvature

As another direction one introduce in the action terms containing Ricci and Riemann tensors like

$$R_{\mu\nu} \mathcal{F} R^{\mu\nu} \text{ and } R_{\mu\nu\alpha\beta} \mathcal{F} R^{\mu\nu\alpha\beta}$$

In a local action in 4 dimensions Riemann tensor squared could be dropped due to the fact that Gauss-Bonnet term is a topological term. In our case, however, even in 4 dimensions both such terms make sense because the presence of an operator in the middle renders them non-trivial. Action exactly of this type is considered in [8] and looks like

$$S = \int d^4x \sqrt{-g} \left(\frac{M_P^2}{2} R + a_1 R \mathcal{F}_1 R + a_2 R_{\mu\nu} \mathcal{F}_2 R^{\mu\nu} + a_3 R_{\mu\nu\alpha\beta} \mathcal{F}_3 R^{\mu\nu\alpha\beta} + \Lambda \right)$$

such an action may be a very interesting generalization, it also surely admits inclusion of higher degrees of curvature tensors but the analysis is highly non-trivial because of the fact that covariant derivatives acting on second and fourth rank tensors involve the corresponding number of metric connections thus making the variation w.r.t. the metric very tedious.

The primary question which must be answered here is the formulation of the ghost free condition in a non-trivial (i.e. non-Minkowski) background. Also it may be interesting to find whether it is possible to reformulate the latter action in a simpler way by means of scalar fields like it was done in the previous Section in case of only the scalar curvature.

6 Outlook

Even though we understand how the background solutions can be constructed in such non-local models and really can tame them in various regimes and even construct exact analytic solutions in specific cases there are major issues one must analyze one by one in each particular setup.

First it is important to guarantee that the perturbative spectrum does not contain ghosts. This is exactly the place where non-local operators do their job. Second, one must answer the question are the perturbations well behaved? To do this the best way is to compute the second variation of the action but this task is quite difficult for non-trivial (i.e. non-constant curvature) solutions. So

far it seems possible to do this either for Minkowski or de Sitter backgrounds. Third and not yet explored is the question: do the obtained solutions represent general behavior of a model or they are just specific measure zero configurations.

There is a natural hope that reformulation of non-local gravity models in terms of p -adic like action may shed more light since p -adic theories are more explored to the moment. It would be interesting to see more specifically the correspondence in between of these two classes of models. On this way action (6) looks as the most doable forthcoming project in developing non-local models since it has clear reformulation in terms of an additional scalar field. A detailed analysis of models based on action (6) is the goal of the upcoming paper [9].

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